NATIONAL COMPREHENSIVE CENTER FOR TEACHER QUALITY

Preparation of Effective Teachers in Mathematics

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A TQ Connection Issue Paper on Applying the Innovation Configuraton to Mathematics Teacher Preparation

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CONTENTS

PURPOSE AND OVERVIEW

The purpose of this paper is to apply the recommendations of the National Mathematics Advisory Panel (2008) to the practice of preparing and developing highly effective mathematics teachers. This paper describes the Innovation Configuration for Effective Teaching and Learning of Mathematics. The Innovation Configuration is offered as a means of evaluating and aligning teacher preparation and professional development activities to promote stronger learning of mathematics in accordance with recent recommendations of the National Mathematics Advisory Panel (2008). The Innovation Configuration provides a set of actionable recommendations for all teacher educators (i.e., preservice teacher educators, inservice professional development providers, school and district leaders) in applying the panel's recommendations to bolster the quality of mathematics teaching and learning in schools.

INTRODUCTION

The Imperative: Improve the Quality of Mathematics Teaching and Learning

Mathematics education has gained significant momentum as a national priority and important focus of school reform (National Mathematics Advisory Panel, 2008). In the United States, student achievement in mathematics, although improving (Slavin, Lake, & Groff, 2009), remains alarmingly low in comparison with other nations (Mullis, Martin, Gonzalez, & Chrostowski, 2003; National Assessment of Educational Progress [NAEP], 2007; Thomson, Cresswell, & De Bortoli, 2004). Only 32 percent of eighth graders scored "proficient" on the U.S. National Assessment of Educational Progress in 2007. Moreover, achievement gaps persist between Caucasian students and students of color, with 42 percent of Caucasian students in eighth grade scoring "proficient" in mathematics, as compared with 11 percent of African American and 15 percent of Hispanic students (NAEP, 2007).

Central to raising student achievement in mathematics is improving the quality of mathematics teaching. Students who receive high-quality instruction experience greater and more persistent achievement gains than their peers who receive lower-quality instruction (Rivkin, Hanushek, & Kain, 2005; Wright, Horn, & Sanders, 1997). Rivkin et al. found that students who were taught by a highly effective teacher achieved a gain of 1.5 grade equivalents during a single academic year, whereas students enrolled in classes taught by ineffective teachers gained only 0.5 grade equivalents in the same year. Moreover, the effects of high-quality instruction on the academic achievement of disadvantaged students are substantial enough to counteract the host of familial and social conditions often found to impede student achievement (Rivkin et al., 2005). To put it differently, teachers are critical determinants of student learning and educational progress and thus must be well trained to use effective teaching practices.

Working Toward a Solution: Teacher Education as Continuous and Sustained Teacher Learning

For the purpose of this paper, we conceptualize teacher education as preparation and development in the practice of teaching, which includes both preservice teacher preparation and inservice professional development. A longstanding and problematic tradition in teacher education is the treatment of teacher preparation and inservice professional development as disparate phases of a teacher's career continuum. A more productive view holds that teacher preparation, induction, and professional development are critical phases of teacher learning rather than fragmented programs, institutions, or policy levers each governed by its own mandates, beliefs, and practices. Revisioning teacher education as sustained and continuous teacher learning that begins at the preservice stage and continues throughout a teacher's career is critical to the advancement of teaching and learning in our schools.

INNOVATION CONFIGURATION ON EFFECTIVE PRACTICES FOR MATHEMATICS TEACHING AND LEARNING

A relevant question to those committed to building sustained programs to support and maximize teacher efficacy is, What are the characteristics of excellent mathematics teaching? That is, what actions should effective teachers take to ensure that students optimally learn mathematics and what skills and resources should training programs provide to ensure teachers know how to take those actions? The Innovation Configuration on Effective Practices for Mathematics Teaching and Learning (referred to hereafter as the Innovation Configuration) provides a summary of the essential components of mathematics instruction and can be used to evaluate teacher preparation and professional development programs. The Innovation Configuration is printed in the Appendix.

Innovation configurations have been used for at least 30 years in the development and implementation of educational innovations and methodologies (Hall & Hord, 1987; Roy & Hord, 2004). They most often have been used as professional development tools to guide implementation of an innovation within a school and to facilitate the change process. Innovation configurations also have provided a form of self-assessment and reflection and can be used in program evaluation as a means of determining the degree to which educational policies are implemented within coursework and supervised field experiences.

Innovation configurations typically are arranged in tables that have two dimensions: one specifying the key principles, and the other specifying levels of implementation (Hall & Hord, 1987; Roy & Hord, 2004). The essential components of the innovation or program appear in the rows of the table's far-left column, along with descriptors and examples to guide application of the criteria to program coursework, standards, and classroom practice. The second dimension is the *degree of implementation*. In the top row of the table, several levels of implementation are specified. For example, no occurrence of the essential component is the lowest level of implementation and might be assigned a score of zero. Increasing levels of implementation are assigned progressively higher scores.

The Innovation Configuration described in this paper is designed to improve mathematics teacher education, which, in turn, may lead to improvement in student achievement in mathematics. The Innovation Configuration can be used to examine the similarities, differences, and gaps among teacher preparation programs and promote the coherence, continuity, and efficacy of teacher preparation to teach mathematics. Innovation Configuration results provide credible information on current teacher preparation practices that can be used as the basis for policy and program changes in teacher preparation programs at the state and university levels.

COMPONENTS OF THE INNOVATION CONFIGURATION FOR MATHEMATICS

Key components of effective mathematics instruction addressed by the Innovation Configuration are the following:

- Subject-matter knowledge in mathematics (or the teacher's knowledge of the content being taught)
- Mathematics topics for student mastery
- Knowledge about how to most effectively teach mathematics (or the teacher's knowledge and use of effective instructional strategies in teaching mathematics)

The following sections briefly describe the components of the Innovation Configuration that should be addressed by mathematics teacher preparation and professional development programs. Preparing teacher candidates in these core competencies can result in more effective instructional processes and outcomes and the advancement of student learning in mathematics (National Mathematics Advisory Panel, 2008).

Subject-Matter Knowledge in Mathematics

> **Research on the relationship between teachers' mathematical knowledge and students' achievement supports the importance of teachers' content knowledge in student learning.**
>
> *—National Mathematics Advisory Panel (2008, p. xxi)*

Research on mathematics teaching suggests that many teachers do not possess the requisite subject-matter knowledge to implement high-quality instruction (Ball, 1990; Ball & Bass, 2000; Ball & Cohen, 1999; Hill, Schilling & Ball, 2004; Ma, 1999; National Commission on Teaching and America's Future, 1996). The National Mathematics Advisory Panel (2008) underscores the need for teachers to know mathematics for teaching in order to teach effectively:

> Teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. (p. 38)

The logic herein is that teachers who possess strong mathematical knowledge at a greater depth and span are more likely to foster students' ability to reason, conjecture, and problem-solve, while also being able to more accurately diagnose and address students' mathematical (mis)conceptions and computational (dys)fluencies (Kilpatrick, Swafford, & Findell, 2001). Two challenges have been associated with ensuring that teachers have the adequate content knowledge to teach mathematics effectively. First, because mathematics education research has been fraught with philosophical differences, defining the content or subject matter that teachers should master has been a matter of some debate (National Council of Teachers of Mathematics, 2006; National Mathematics Advisory Panel, 2008). The National Mathematics Advisory Panel Task Group on Teachers and Teacher Education (Ball, Simons, Wu, Whitehurst, & Yun, 2008) commented, “defining a precise body of mathematical knowledge that would effectively serve teachers and would guide teacher education, professional development, and policy has proved challenging” (2008, p. 5-x). Second, the use of indirect indicators or proxies for teacher knowledge, such as teacher certification, coursework, and teacher licensing exams, rather than more robust and direct measures of teachers’ mathematical knowledge, has made the study of content knowledge and its link to student learning difficult (Hill, Rowan, & Ball, 2005).

Despite these challenges, research on the relationship between teachers’ mathematical knowledge and student achievement has offered some evidence of the impact of mathematical knowledge on teaching effectiveness and student learning. Most research in this area has focused on secondary mathematics teaching and suggests general positive influences of teachers’ studying mathematics on student achievement (Goldhaber & Brewer, 1997, 2000; Hawkins, Stancavage, & Dossey, 1998; Monk, 1994; Monk & King, 1994). These positive effects, however, varied by skill level of student (e.g., whether the students were enrolled in advanced or remedial classes) and number of undergraduate mathematics courses taken by the teacher (Monk, 1994). Although results in studies of teachers’ mathematical knowledge and student achievement are mixed, the extant evidence does suggest teachers’ knowledge of mathematics content is a contributor to instructional quality and student achievement (National Mathematics Advisory Panel, 2008; Wilson, Floden, & Ferrini-Mundy, 2001). The National Mathematics Advisory Panel’s Task Group on Teachers and Teacher Education concluded

> **Research on the relationship between teachers’ mathematical knowledge and students’ achievement supports the importance of teachers’ content knowledge in student learning.** However, because most studies have relied on proxies for teachers’ mathematical knowledge (such as teacher certification or courses taken), existing research does not reveal the specific mathematical knowledge and instructional skills needed for effective teaching, especially at the elementary and middle school level. (Ball et al., 2008 p. 5-xi, emphasis added)

These findings suggest that preparation and professional development programs for mathematics teachers should emphasize the mathematical topics for student mastery that the National Mathematics Advisory Panel identified and ensure that teachers possess a strong knowledge base in the topics that students must master. The next section of this paper summarizes skills and topics that the National Mathematics Advisory Panel identified as being most important for students to master.

Mathematics Topics for Student Mastery

> **Proficiency with whole numbers, fractions, and particular aspects of geometry and measurement should be understood as the Critical Foundations of Algebra.**
>
> *—National Mathematics Advisory Panel (2008, p. 18)*

> **Teacher education programs and licensure tests for early childhood teachers, including all special education teachers at this level, should fully address the topics on whole numbers, fractions, and the appropriate geometry and measurement in the Critical Foundations of Algebra, as well as the concepts and skills leading to them.**
>
> *—National Mathematics Advisory Panel (2008, p. 19)*

Undergirding recommendations for systematic mathematics education reform is the National Mathematics Advisory Panel's findings on the critical importance of student mastery in the critical foundations of algebra and the major topics of school algebra. Algebra has long been identified as the gatekeeper to academic achievement and educational attainment (National Council of Teachers of Mathematics, 2001). Students who successfully complete high school algebra are more likely to graduate from high school and college and earn a higher salary (Rivera-Batiz, 1992). Student proficiency in whole numbers, fractions, and aspects of geometry and measurement facilitate student understanding and advancement in algebra (National Council of Teachers of Mathematics, 2000, 2006) To foster students' mastery in these core mathematical domains and to promote more complex mathematical understandings of concepts, all teachers of mathematics should master the major topics of school algebra identified in the National Mathematics Advisory Panel Report (2008, p. 16) and shown in Table 1.

Table 1. Major Topics of School Algebra

Symbols and Expressions

- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations

- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations

- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability

- Combinations and permutations as applications of the binomial theorem and Pascal's triangle

Teachers must deepen their knowledge of this content, including the proper sequencing and closure of these topics and the topics that precede and follow them. Moreover, as emphasized in the National Mathematics Advisory Panel's recommendations, this knowledge is not for general mathematics teachers alone—**all** teachers of mathematics, **including early childhood and special education teachers,** should demonstrate knowledge of the critical foundations of algebra, actively and collectively share responsibility for student learning from school entry through high school algebra, and understand topic sequencing and cognitive demands from early to advanced learning of mathematics.

Knowledge About How to Teach Mathematics

> **It is not just what mathematics teachers know, but how they know it and what they are able to mobilize mathematically in the course of teaching.**
>
> *—Ball (2000, p. 243)*

> **This knowledge is explicitly multidimensional.**
>
> *—Hill, Ball, & Schilling (2008, p. 396)*

The notion of a specialized knowledge base for teaching has been in existence for more than twenty years. In 1986 Lee Shulman identified a specialized form of teacher knowledge necessary for the practice of effective teaching: *pedagogical content knowledge*. Shulman defined pedagogical content knowledge as the knowledge and means of "representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). Shulman argued that content-absent pedagogy is problematic to the classroom teacher, who relies on content knowledge to deliver instruction and advance student learning.

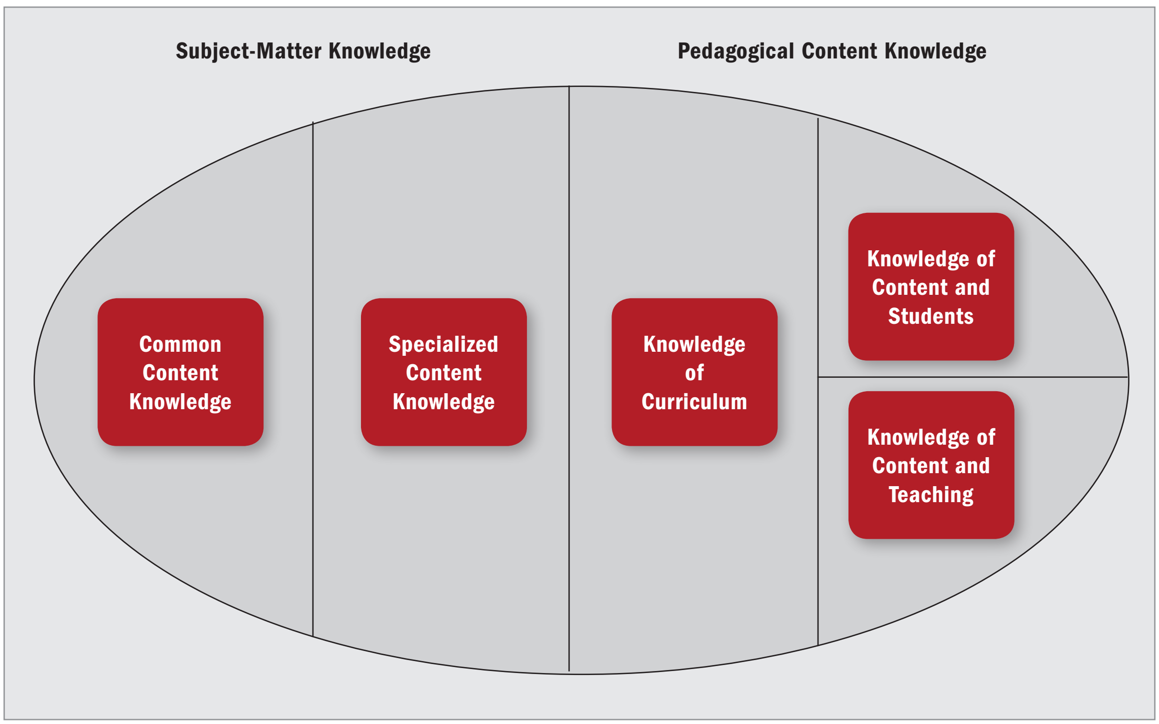
Knowledge about how to teach mathematics differs in important ways from content knowledge possessed by professionals in other mathematics-related disciplines (Hill, Ball, & Schilling, 2008). Mathematics teachers must know not only the content they teach, but also how students' knowledge of mathematics is developed and structured; how to manage internal and external representations of mathematical concepts; how to make students' understanding of mathematics visible; and how to diagnose student misunderstandings and misconceptions, correct them, and guide them in reconstructing complex conceptual knowledge of mathematics (Ball, Lubienski, & Mewborn, 2001; Cohen & Hill, 2000; Darling-Hammond, 1999; Fennema & Franke, 1992). Moreover, teachers must understand how students reason and employ strategies for solving mathematical problems and how students apply or generalize problem-solving methods to various mathematical contexts (Cobb, 1986). The use of language, construction of metaphors and scenarios appropriate to teaching mathematical concepts, and understandings and use of curricular resources in the practice of teaching constitute a knowledge base for teaching that is specific to and grounded in the teaching of mathematics. These understandings represent the "specialized content knowledge" (Hill & Ball, 2004, p. 333) effective teachers of mathematics possess (Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005). Moreover, this knowledge transcends traditional subject-matter knowledge—or "common knowledge of content"—

and knowledge of classroom practices for teaching mathematics. Rather, knowledge of mathematics for teaching is embedded in the practice of teaching mathematics:

> in mathematics, how teachers hold knowledge may matter more than how much knowledge they hold. In other words, teaching quality might not relate so much to performance on standard tests of mathematics achievement as it does to whether teachers' knowledge is procedural or conceptual, whether it is connected to big ideas or small bits, or whether it is compressed or conceptually unpacked....student learning might result not only from teachers' content knowledge but also from the interplay between teachers' knowledge of students, their learning, and strategies for improving that learning. (Hill & Ball, 2004, p. 332)

Refinements in conceptualizing knowledge of mathematics for teaching are currently underway. Hill and her colleagues have developed a conception of "knowledge of content and students" (Hill, Ball, & Schilling, 2008), which is one component of pedagogical content knowledge yet differs from "knowledge of content and teaching" and "knowledge of curriculum" (Hill, Ball, & Schilling, 2008, p. 377).

Figure 1. Multidimensional Nature of Knowledge for Teaching Mathematics



Source: Adapted from Hill, Ball, & Shilling, 2008, p. 377

Although this concept requires further development and clearer investigative results, it is critical that teacher educators and professional development providers keep students and their learning at the center rather than periphery and focus deliberately on student learning processes and outcomes when designing teacher learning activities (Messick, 2005). Beginning with what mathematics students know and our understandings of how they know it is an effective approach to bolstering the quality of instruction and student learning (Carpenter & Fennema, 1992; Carpenter, Fennema & Franke, 1996).

Instructional Methods for Teaching Mathematics

> **All-encompassing recommendations that instruction should be entirely "student centered" or "teacher directed" are not supported by research....High-quality research does not support the exclusive use of either approach.**
>
> *—National Mathematics Advisory Panel (2008, p. 45)*

> **[T]he Panel recommends regular use of formative assessment, particularly for students in the elementary grades....for struggling students, frequent (e.g., weekly or biweekly) use of these assessments appears optimal, so that instruction can be adapted based on student progress.**
>
> *—National Mathematics Advisory Panel (2008, p. 47)*

> **The Panel recommends that students with learning disabilities and other students with learning problems receive, on a regular basis, some explicit systematic instruction that includes opportunities for students to ask and answer questions and think aloud about the decisions they make while solving problems. This kind of instruction should not comprise all mathematics instruction students receive.**
>
> *—National Mathematics Advisory Panel (2008, p. 48–49)*

The precise characteristics of effective mathematics instruction have been debated for decades (Kilpatrick et al., 2001). Scholars and educators have grappled with tensions between teacher-directed instruction and student-centered constructivist approaches, each offering evidence for its effectiveness in achieving some mathematical outcome (e.g., improvement in teacher facilitation of classroom discussion, development of classroom mathematical norms, student learning of particular topics).

While the National Mathematics Advisory Panel (2008) identified a set of effective instructional practices, others have argued that certain approaches recommended as producing desired teacher and student outcomes were not included (Lobato, 2008). Our purpose is not to resolve this debate. The purpose of this section is to identify instructional practices that allow all students access to complex mathematical ideas and meaningful mathematical activities that support the advancement of mathematical competence. These instructional practices are enumerated in the Innovation Configuration, which serves as a planning and evaluation tool for mathematics teachers from preservice preparation to inservice professional development.

Effective instructional practices for equitable student access to mathematics learning are encapsulated within the following areas: task, classroom discourse or discussion, and assessment. Each section that follows explicates essential teaching competencies for each domain.

Task: Designing Meaningful Mathematical Activities for Student Learning

> **Students' academic work in school is defined by the academic tasks that are embedded in the content they encounter on a daily basis. Tasks regulate the selection of information and the choice of strategies for processing that information....Students will learn what a task leads them to do, that is, they will acquire information and operations that are necessary to accomplish the tasks they encounter.**
>
> *—Doyle (1983, p. 162)*

Theories of mathematical learning suggest that the nature of classroom activities, or "tasks," affects students' abilities to learn mathematics with understanding (Doyle, 1983; Hiebert & Wearne, 1993). Hiebert and Carpenter (1992) contend that the structure and depth of knowledge depends on learners' prior knowledge and their ability to access it and connect complex mathematical ideas to one another within a broader network of understanding. Students with connected knowledge structures are better equipped to engage in reasoning and problem solving and better primed to transfer learning and adapt understandings to new contexts (Bransford, Brown, & Cocking, 1999). Therefore, the mathematical tasks in which students engage should facilitate and support students' conceptual understanding of mathematics, fostering deep connections among mathematical ideas (Hiebert & Carpenter, 1992).

The majority of time students spend in the classroom is on tasks (Hiebert & Wearme, 1993), and thus the quality thereof and their potential to advance students' thinking is paramount. Instructional planning is an important precursor to effective teaching, and when designing instructional tasks, teachers should identify the following:

- What are the important mathematics procedures and concepts for students to understand and why?
- At what level and in what ways do students understand the mathematics prior to the task? In other words, what is students' prior knowledge (including both their knowledge of the topic and at what cognitive level)?
- Is the content of the task identified in standards for teaching mathematics? What is the relationship of this content to the critical foundations of algebra and major topics of school algebra (National Mathematics Advisory Panel, 2008)?
- How does the task move students' mathematical understanding forward? How does the task foster students' construction of new understandings of mathematics?
- How does the task provide all students access to mathematics by building on their prior knowledge? How does the task offer multiple entry points to engage in higher-order mathematical thinking?
- How does the task offer students opportunities to experience multiple representations of concepts? In what ways does the task allow students to employ multiple problem-solving strategies in varied contexts?

How teachers design tasks for students has many implications for if and how students learn mathematics (Boston & Wolf, 2006; Stein, Grover, & Henningsen, 1996). High-quality tasks are those that are well matched to a student's level of understanding, providing a balance between challenging the student and limiting frustration and inadvertent misunderstandings and errors.

High-quality tasks foster students' abilities to reason, solve problems, and conjecture (Matsumura, Slater, Junker, Peterson, Boston, et al., 2006). When tasks are too narrowly conceived, for example, requiring a student to memorize a formula or procedure without reference to prior knowledge and potential applications or variations of that procedure, students may learn a disconnected skill that does not contribute to the student's broader mathematical competence.

Developing fluency with operations and computations is critically important to students' mastery of mathematics (National Mathematics Advisory Panel, 2008); tasks should ensure that students master standard algorithms and procedures, but at the same time they also should come to understand when, how, and in what ways to use those procedures to solve problems of different kinds. In sum, teachers must know how to design meaningful instructional tasks that are aligned to their students' prior knowledge and hold the potential to deepen students' conceptual understanding of mathematics. For example, when young children learn to count fluently forward and backward, this skill should be connected to the concept of cardinality or quantity comparisons and teachers may use measurement tasks to illustrate differences between numerical rankings. As appropriate to the activities used, connections should be made to counting, combining and recombining sets, and early understanding of the principles of addition and subtraction.

Wiggins and McTighe (2001) recommend that teachers establish clear learning goals and clear criteria by which they are assessed. Learning goals can be used to construct a progression of learning activities. In mathematics, learning progressions should align with the National Mathematics Advisory Panel's major topics of school algebra, but they need to be more detailed so as to delineate skills in a coherent, sequenced manner and so as to establish transparent targets for assessment (Heritage, 2008). Brief assessment of students' mastery of key concepts and skills in a learning progression increases teachers (and learners') awareness of any gaps between the desired learning goal and the students' current knowledge and skill (Ramaprasad, 1983; Sadler, 1989). When teachers know where the students are experiencing difficulty, they can use that information to make the necessary instructional adjustments, such as reteaching, allowing extra opportunities for practice, providing instruction in small groups, or changing the method or type of instruction. More frequent assessment provides teachers and students with more immediate feedback on performance and allows self-evaluation on teaching practices and student performance.

Classroom Discussion for Identifying Mathematical Understanding and Advancing Mathematical Knowledge and Skill

> **The nature of mathematical discourse is a central feature of classroom practice. If we take seriously that teachers need opportunities to learn from their practice, developing mathematical conversations allows teachers to continually learn from their students. Mathematical conversations that center on students' ideas can provide teachers a window into students' thinking in ways that students' individual work cannot do alone.**
>
> *—Franke, Kazemi, and Battey (2007)*

Teachers should provide opportunities for learners to share their thinking and problem-solving processes, justify and formulate conjectures publicly, and evaluate multiple solution strategies (Franke, Kazemi, & Battey, 2007; Lampert, 2001; Strom, Kemeny, Lehrer, & Forman, 2001). To ensure high-quality mathematical discourse or discussion, teachers must structure conversations and communication around particular problems and solution methods (Webb, Franke, Ing, Chan, De, et al., 2008; Wood, 1998), engage all students in classroom discussion, including struggling students (Empson, 2003; Moschkovich, 2002; Yackel, Cobb, & Wood, 1991), and respond to a variety of student responses in ways that illuminate and amplify essential mathematical understandings (Franke, Kazemi, & Battey, 2007; Lampert, 1990; O'Connor & Michaels, 1993). The enactment and facilitation of high-quality, content-rich classroom discussion is not easy. It demands that teachers attend to students' thinking both individually and collectively, aligning one student's thinking with another's while also aligning student talk with the content and essential mathematical ideas (Ball, 1997; Lampert & Blunk, 1998; Wilson & Ball, 1996).

Researchers have identified several discursive instructional practices shown to promote learning of mathematics. First, teachers may use a spontaneous, unplanned classroom opportunity to prompt student talk about problem-solving methods, or they may invent problem-solving circumstances ripe with opportunities for student talk about solution methods and conjectures (Yackel, Cobb, & Wood, 1991). Further, it is recommended that teachers provide ample time for students to explore and work on a problem and develop their own ideas for solution strategies and explain them. Thus, teachers should press for student explanations and justifications of their methods and answers. It is important that teachers avoid an assumption-oriented classroom in which they assume students have mastered a concept or skill on the basis of a single answer without adequate justification for it (e.g., Silver & Smith, 1996, Webb et al., 2008). Teachers should also avoid funneling (Wood, 1998) questions in ways that inhibit active student thinking about mathematics, and similarly, teachers should avoid giving students a solution method without allowing time for students to do the intellectual work (Silver & Smith, 1996).

Teachers should model thinking aloud, explaining and clarifying the mental processes used to solve problems and providing justifications and reasoning behind why particular methods were used (Ball, 1993; Lampert, 1990; Rittenhouse, 1998; Webb et al., 2008). Explicit and systematic instruction of this kind is particularly important for struggling students and students with disabilities (Gersten, Beckmann, Clarke, Foegen, Marsh, et al., 2009; National Mathematics Advisory Panel, 2008). To increase student participation, teachers should press for students to explain why and how problem-solving procedures were successful rather than simply calling for the recitation of a procedure. Teachers may use such prompts as "Can you explain to [student] what you did?" "How do you know?" "What makes you think so?" "Can you say more about [your method]?" or "Why did you [identify or name method]?" (adapted from Webb et al., 2008). In addition, teachers can assist students in understanding the extent to which particular problem-solving strategies generalize to other contexts by using prompts similar to the following: "Is there a different way to arrive at this answer?" or "What if you [did this differently]—would you reach the same conclusion?" Teachers' questioning techniques are important contributors to the quality of instruction and student learning, so it is important to prepare teachers to employ a range of questioning techniques for particular purposes, illustrating how the question engages students' thinking and allows students to make connections between problem solutions and important mathematical concepts like equivalence.

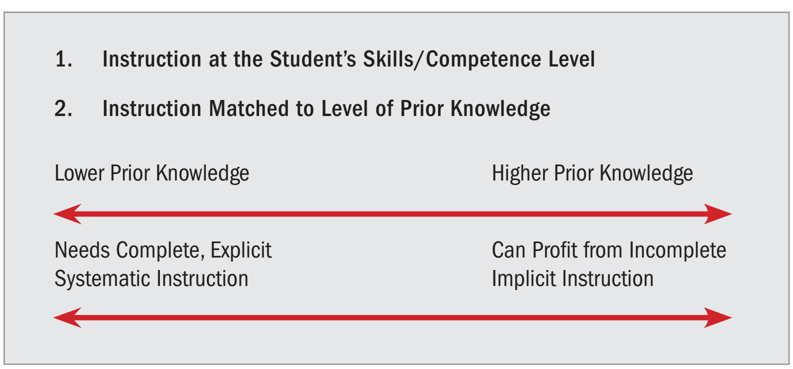
Of particular importance in classroom discussion is how teachers recognize errors in students' solution strategies and understandings of mathematical ideas, and how teachers make productive use of such errors. Errors can be detrimental to learning when they are not responded to effectively (Touchette & Howard, 1984). When errors are too frequent, they may interfere with acquisition of a new skill, and when they are repeated they can become established and difficult to eradicate. When new information and ideas are being introduced to students, more intensive checking of student understanding is necessary so that teachers can detect and respond to errors in ways that facilitate (rather than interfere with) student learning. Teachers can use errors that become apparent during classroom discussion as instructional leverage to make explicit the key concepts and tools of inquiry needed to understand and apply a mathematical principle. Errors can occur spontaneously during instruction but may be anticipated to occur at higher rates when new skills or tasks are introduced. Properly responding to errors provides critical entry points for teachers to facilitate learning for all students.

Use of Assessment to Match Instructional Strategies to Student's Skill Level and Prior Knowledge

A hallmark for high-quality mathematics instruction is the ability to use a continuum of instructional strategies to reach the diverse needs of learners in the classroom. The National Mathematics Advisory Panel (2008) stated that "all-encompassing recommendations that instruction be entirely 'student centered' or 'teacher directed' are not supported by research… and should be avoided" (p. 45). Teachers should understand the balance between student-centered and teacher-directed instruction and know how to use data on students' prior knowledge to identify the instructional strategy that will be of greatest benefit to the student at that student's level of proficiency or understanding. (See Figure 2.)

Figure 2. Relationship of Prior Knowledge and Effective Instruction

Adapted from Cronbach & Snow, 1977, and Vaughn, Gersten, & Chard, 2000



Source: Figure adapted from Reschly, Holdheide, & McGraner, in press.

For instance, research has long supported explicit, direct, systematic instruction for the teaching of content to students with disabilities (Gersten & Chard, 1999; Gersten et al., 2009). The National Mathematics Advisory Panel (2008) has recently recognized the importance of explicit instruction as an intervention with students who are not proficient in mathematics. Gersten et al. (2009) in particular recommended that students receiving mathematics interventions in elementary and middle grades receive approximately 10 minutes of direct instruction daily "to build quick retrieval of basic arithmetic facts" (p. 11). The Gersten et al. also recommended, however, that teachers "provide students with opportunities to solve problems in a group and communicate problem-solving strategies" (p. 11). It is important to prepare teachers with a broad repertoire of instructional practices that ensure all students learn mathematics with understanding and have multiple entry points to access complex mathematics. To identify the type of instruction best suited to an individual student's needs, formative assessment of mathematics proficiency is needed.

Assessing Students' Understanding of Mathematics

The regular use of formative assessment has been documented to mitigate and prevent mathematical difficulties and improve student learning when used to inform instruction (Clarke & Shinn, 2004; Fuchs, 2004; Lembke & Foegen, 2005; Skiba, Magnusson, Marston, & Erickson, 1986). The Council of Chief State School Officers published a widely cited definition of formative assessment in 2006: "Formative assessment is a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes" (p. 1). Formative assessments may be designed and enacted in many ways, including "on-the-fly" and "planned-for interactions," as well as "curriculum-embedded assessments" (Heritage, 2007, p. 143). For example, in the course of a lesson, a teacher may overhear a student's misconception or error and then spontaneously and promptly adjust the instruction to address the misconception and reteach (i.e., "on-the-fly"). Moreover, a teacher may deliberately plan to elicit students' thinking during the lesson, providing a critical assessment point at which the teacher can use the gathered evidence to adjust instruction. Formative assessment should include methods for identifying students' progress, providing feedback, involving the student in assessing understanding, and tending to learning progressions (Heritage, 2007). It is critical that both the student and the teacher are active participants in identifying growth and reflecting on learning behaviors—critical ingredients to moving achievement forward.

Curriculum-based measurement is a type of formative assessment. Curriculum-based measurements are brief, direct, systematic assessments of student performance on tasks that predict future learning success on the intended general outcomes of instruction. In mathematics, instruction guided by frequently administered curriculum-based measurement has been demonstrated to enhance mathematics achievement (Fuchs, Fuchs, Hamlett, & Stecker, 1990; Stecker & Fuchs, 2000).

In selecting curriculum-based measurement tools for use in the classroom, teachers must have an understanding of the intended learning outcomes and the sequence (and time period) within which attaining the outcomes can be expected. Generally, the number of tasks assessed on a curriculum-based measurement is proportionate with the time within which learning is to be evaluated; fewer skills are associated with shorter duration of instruction. The approach of measuring narrowly defined skills and tasks to judge student mastery of that specific skill has been referred to as mastery measurement (Fuchs & Deno, 1994), which can be highly useful for teachers and others who wish to make judgments about short-term instructional effects (for example, during intervention planning or in determining when to advance task difficulty). On the other hand, to model student growth over time and toward the more general outcomes of instruction belies the need for broader content sampling and can be used to evaluate whether short-term instructional adjustments are effective in placing students on track to attain the broader outcomes of instruction (VanDerHeyden, 2005). Formative assessment should be recognized as a key professional skill for teachers and given high priority in training and professional development programs. Through appropriate instruction and well-crafted learning experiences, teachers and teacher candidates should have a solid foundation of knowledge related to formative assessment (developing learning progressions or skill sequences, assessing general outcomes via curriculum-based measurement, and skill and subskill mastery via mastery measurement) and an understanding of the various types of formative assessment (e.g., curriculum-based measurement, mastery measurement), including their distinct features and advantages. Teachers also must be adept in evaluating the merits of certain assessments and their suitability for answering certain questions germane to their instruction. Once measures are selected and administered, teachers must be adept at using the resulting data to inform instruction. Teacher preparation programs and professional development activities also should provide instruction and opportunities for application in which teachers can make the connection between the use of formative assessment measures and the decisions that must be made within response-to-intervention frameworks (e.g., determining the need for classwide, small-group, or individual supplemental intervention, selecting intervention facets that will produce learning gains, and evaluating intervention effects) (VanDerHeyden, 2009).

In the early years (PK through Grade 1), assessment of student mastery (and progress toward mastery) of number sense should be emphasized. Such assessments include one-to-one object correspondence in counting, counting in sequence, counting on, counting backward, number identification and naming, identifying larger and lesser quantities, and completing missing numbers within number sequences (Clarke & Shinn, 2004; Methe, Hintze, & Floyd, 2008; VanDerHeyden, Broussard, Fabre, Stanley, Legendre, et al., 2004; VanDerHeyden, Witt, Naquin, & Noell, 2001). As students progress through the primary grades, direct assessment of the fluency (accuracy and speed) with which they compute basic facts and more complicated operations (e.g., finding least common denominator) become important assessment targets to identify where progress toward important benchmarks is lagging and intervention may be warranted (see VanDerHeyden, 2009, for suggested universal screening measures for mathematics Grades K through 8). Direct measures of the extent to which students demonstrate conceptual understanding of mathematical principles can be directly assessed and contribute information that is distinct from computation and operational fluency (Fuchs, Fuchs, Bentz, Phillips, & Hamlett, 1994) with implications for instructional planning (Fuchs, Fuchs, Prentice, Burch, Hamlett, et al., 2003).

CONCLUSION

There is a science of mathematics education that includes clear implications for curricula content (what is taught) and instructional practices (how it is taught) (Gersten et al., 2009; National Mathematics Advisory Panel, 2008). The Innovation Configuration was based on these summary reports and the broader literature in mathematics education and teaching. The Innovation Configuration offers a set of quantifiable indicators of instructional excellence in mathematics that are related to improved achievement in mathematics that can be used to improve teacher competencies and student achievement.

APPENDIX: Mathematics Innovation Configuration

	Code = 0	Code = 1	Code = 2	Code = 3	Code = 4	Rating
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria specified, from 0 to 4. Score and rate each item separately. Descriptors and examples are bulleted below each component.	**No evidence** that the concept is included in class syllabi	**Concept mentioned** in class syllabi	Concept mentioned in syllabi and required **readings and tests and/or quizzes**	Concept mentioned with readings, tests, and **assignments, projects for application** • Observations Lesson plans • Classroom demonstration • Journal response	Concept mentioned in class, required reading, tests, projects, assignments, **and teaching with application and feedback** • Fieldwork (practicum) • Tutoring	Rate each item as the number of the highest variation receiving an X under it.
Subject-Matter Knowledge in Mathematics • College-level course-taking in mathematics content consistent with grade level(s) taught, including content both preceding and following level(s) taught • Strong knowledge base of the mathematical topics recommended by the National Mathematics Advisory Panel (2008)						
Mathematic Topics of Student Mastery • Pedagogical and curricular knowledge of mathematics: ▪ Topics of whole numbers, fractions, and geometry (critical foundations of algebra) ▪ Symbols and expressions, linear equations, quadratic equations, functions, algebra and polynomials, and combinatorics and finite probability (major topics of school algebra) • Selection, sequencing, and closure of topics and the appropriate cognitive demand(s) of the task(s) that precede and follow						

	Code = 0	Code = 1	Code = 2	Code = 3	Code = 4	Rating
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria specified, from 0 to 4. Score and rate each item separately. Descriptors and examples are bulleted below each component.	**No evidence** that the concept is included in class syllabi	**Concept mentioned** in class syllabi	Concept mentioned in syllabi and required **readings and tests and/ or quizzes**	Concept mentioned with readings, tests, and **assignments, projects for application** • Observations Lesson plans • Classroom demonstration • Journal response	Concept mentioned in class, required reading, tests, projects, assignments, **and teaching with application and feedback** • Fieldwork (practicum) • Tutoring	Rate each item as the number of the highest variation receiving an X under it.
Knowledge of Mathematics for Teaching • Pedagogical and curricular knowledge of mathematics • How students learn mathematics, including common misconceptions of and errors in students' learning of mathematics • Methods to identify and diagnose students' prior knowledge • How/when to employ particular strategies to address students' (mis)understandings • Methods to support the development of conceptually unpacked knowledge (i.e., how to facilitate students' development of connections and understandings of relationships among mathematics concepts) • Conceptual mathematics activities, including but not limited to identifying and explaining patterns, developing conjectures and predictions, testing, proving, generalizing, and refuting.						

	Code = 0	Code = 1	Code = 2	Code = 3	Code = 4	Rating
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria specified, from 0 to 4. Score and rate each item separately. Descriptors and examples are bulleted below each component.	**No evidence** that the concept is included in class syllabi	**Concept mentioned** in class syllabi	Concept mentioned in syllabi and required **readings and tests and/ or quizzes**	Concept mentioned with readings, tests, and **assignments, projects for application** • Observations Lesson plans • Classroom demonstration • Journal response	Concept mentioned in class, required reading, tests, projects, assignments, **and teaching with application and feedback** • Fieldwork (practicum) • Tutoring	Rate each item as the number of the highest variation receiving an X under it.
Effective Instructional Strategies						
Designing Meaningful Mathematical Activities for Student Learning • Selection and design of instructional tasks and mathematics learning activities • Identification of the mathematics to learn for understanding and the connection thereof to school algebra and to the mathematical learning trajectories of students • The use of both ***teacher-directed*** (e.g., direct, systematic instruction with feedback) and ***student-centered*** (e.g., guided inquiry, open-ended tasks) instructional practices appropriate to students' prior knowledge and mathematics learning goals • Explicit teaching of problem-solving processes using external representations and tools (e.g., charts, diagrams, manipulatives) • Designing multiple entry points for student access to mathematical ideas • Use of and facility with multiple and varied solution strategies to solve problems, processes to support students' formulations of conjectures, arguments, proofs, reasoning, and generalizations • Tools to support learning mathematics with understanding, including but not limited to external representations (e.g. charts, graphs, diagrams)						

	Code = 0	Code = 1	Code = 2	Code = 3	Code = 4	Rating
Instructions: Place an X under the appropriate variation implementation score for each course syllabus that meets the criteria specified, from 0 to 4. Score and rate each item separately. Descriptors and examples are bulleted below each component.	**No evidence** that the concept is included in class syllabi	**Concept mentioned** in class syllabi	Concept mentioned in syllabi and required **readings and tests and/ or quizzes**	Concept mentioned with readings, tests, and **assignments, projects for application** • Observations Lesson plans • Classroom demonstration • Journal response	Concept mentioned in class, required reading, tests, projects, assignments, **and teaching with application and feedback** • Fieldwork (practicum) • Tutoring	Rate each item as the number of the highest variation receiving an X under it.
Classroom Discussion • Designing multiple entry points for student access to mathematical ideas • Methods for teacher facilitation of students' mathematical discourse, including : • Thinking aloud • Making connections among students' responses to mathematical problems • Making explicit the steps of problem-solving processes • Resolving discrepant answers • Questioning and clarifying students' thinking • Pressing for mathematical reasoning and explanations						
Assessment of Student Learning • Classwide and supplemental intervention approaches with curriculum-embedded assessment • Construction of formative assessments to identify small increments of learning and growth (e.g. curriculum-based measurement • Use of formative assessments as instructional, learning, and measurement tools • Analysis of assessment and progress-monitoring data, methods of altering instruction, and interventions based on these data						

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ABOUT THE NATIONAL COMPREHENSIVE CENTER FOR TEACHER QUALITY

The National Comprehensive Center for Teacher Quality (TQ Center) was created to serve as the national resource to which the regional comprehensive centers, states, and other education stakeholders turn for strengthening the quality of teaching—especially in high-poverty, low-performing, and hard-to-staff schools—and for finding guidance in addressing specific needs, thereby ensuring that highly qualified teachers are serving students with special needs.

The TQ Center is funded by the U.S. Department of Education and is a collaborative effort of ETS, Learning Point Associates, and Vanderbilt University. Integral to the TQ Center's charge is the provision of timely and relevant resources to build the capacity of regional comprehensive centers and states to effectively implement state policy and practice by ensuring that all teachers meet the federal teacher requirements of the current provisions of the Elementary and Secondary Education Act (ESEA), as reauthorized by the No Child Left Behind Act.

The TQ Center is part of the U.S. Department of Education's Comprehensive Centers program, which includes 16 regional comprehensive centers that provide technical assistance to states within a specified boundary and five content centers that provide expert assistance to benefit states and districts nationwide on key issues related to current provisions of ESEA.

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